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STUDENTS' DIFFICULTIES IN COMPREHENDING MATHEMATICAL WORD PROBLEMS IN ENGLISH LANGUAGE LEARNING CONTEXTS

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ABSTRACT

This article highlights students' difficulties in comprehending word problems especially in English Language Learning (ELL) contexts such as Pakistan. In such a context, students generally grapple with the language and experience difficulty in comprehending word problems. Therefore, it seems reasonable to expect that a student's performance in solving word problems is affected by difficulties in comprehension. Keeping in mind the importance of comprehension in solving word problems, this research aimed to explore and improve teaching and learning practices in the area of mathematical word problems in a private school in Karachi. The findings of the study reveal that students seemed to encounter issues in comprehending word problem statements which resulted in ineffective teaching and learning practices. To deal with these issues, some recommendations have been put forth to revisit teaching and learning practices.

Key words: Mathematical word problems; comprehension; English language learning; teaching and learning practices

1. INTRODUCTION

Word problems are an integral part of the Mathematics curricula. However, children face difficulties in solving mathematical word problems as most of the time they do not comprehend the wording of the problem. In Pakistan, children are often overwhelmed by Word Problems (WPs) not because they cannot solve these but because they do not comprehend the problem statement due to a language barrier. As a result they often wait for the teacher to solve the question in numerical form; otherwise students tend to rely on key words or misinterpret the problem statement and come up with the wrong answer themselves. The interpretation of a problem statement becomes more crucial to understand in the context of developing countries where English is taught as a second or a foreign language and the medium of instruction is bilingual or even multilingual. For instance, competence in the language which is used as a tool in the classroom for communication becomes a prerequisite for English Language Learners (ELLs). Many studies (e.g. Abedi & Lord, 2001; Bernardo, 2002; Cuevas, 1983) have shown that pupils' failure on WPs is due to a lack of linguistic knowledge. This situation becomes even more problematic when the word problem is expressed in the learner's second or third language. Research carried out in New Zealand (Bartin, Chan, King, Neville-Barton & Sneddon, 2005) with students for whom English was a second language concluded that learners experience a disadvantage of between 10-15% in Mathematics as a result of language difficulties. Similar results were found in other ELL and English as a Second Language (ESL) contexts including Pakistan (e.g. Halai, 2001; Jamaluddin, 1999; Khan, 2009). According to Hegarty, Mayer and Monk (1995) two distinct paths are used by students while comprehending text: the direct translation approach and a problem model approach. The former relies on key words rather than forming mental representations of the problems. What research has found is that if students are asked to rely solely on knowing certain key words it can actually detract them from trying to understand the problems (Krick-Morales, 2006). Key words can cause confusion in differentiating between everyday language and mathematical language. For instance, "The mathematical language that we use (symbols, pictures, words, and numbers) is sometimes unique (only used by mathematicians) or is taken from everyday language and turned into something else" (Kotsopoulos, 2007, p. 302). Therefore, the task of comprehending word problems is critical and represents the threshold to successful solutions (Valentin & Sam, 2004).

2. THEORETICAL UNDERPINNING

This research aims to explore students' difficulties in comprehending and solving word problems in an ELL classroom context. It also intends to present the supporting and constraining factors that contribute towards students' understanding of WPs. The role of language comprehension is pivotal in the teaching and learning of Mathematics because understanding mathematical concepts and solving problems primarily depend on the language used in the

process of teaching and learning. For example, Riordin and O'Donoghue (2008) argue that "performance on mathematical word problems is related to language proficiency" (p.58). It has been generally observed that students spend a considerable amount of time trying to understand the problem because they experience difficulty in making sense of WPs. Krick-Morales (2006) notes that, "Word problems in mathematics often pose a challenge because they require that students read and comprehend the text of the problem, identify the question that needs to be answered, and finally create and solve a numerical equation" (p.1). Hence it is challenging to construct meaning by reading a problem statement superficially. According to Orton:

> It is possible to read a story or novel in English in a fairly superficial way, and yet still derive meaning, message and moral. It is even possible to use rapid reading techniques, perhaps skipping sentences or descriptive paragraphs which are clearly not crucial. Non-fiction cannot generally be read in a superficial way without losing details that might be essential and mathematical text comes into this category. (p.133)

Consequently, the role of comprehending the text of the word problem is crucial because it is not only a means of conveying information; rather it is used to interpret the events and phenomenon in a way that provokes students' thinking (Panah, 2000).

Furthermore, WPs are interesting with respect to their role in the teaching and learning of Mathematics because they require the integration of several competencies: language understanding is one of them (Anderson, 2007; Thevenot & Oakhill, 2008). Therefore, without understanding the language of WPs it is difficult to initiate the process of solving the problem. As a result, students' opportunities for success in solving the problem decrease. Garderen (2004) endorses this notion when he notes that "solving mathematical word problems is often hindered by the student's failure to comprehend the problem" (p.225). Further, comprehension becomes even more problematic for ELL learners due to a lack of proficiency in the English language. Bautista, Mitchelmore and Mulligan (2009) assert that "learning mathematics in general, and solving word problems in particular, poses difficulties, given that large-scale assessments show that many students are not proficient in the language" (p.729). Likewise, research (e.g. Bernardo, 2002; Oviedo, 2005) has shown similar findings in that students' difficulties in comprehending WPs is due to a lack of understanding the language of the problem. Students tend to solve problems easily if presented with a numerical version rather than as words; however, they may fail to solve WPs even though they can solve corresponding problems given in purely numerical format. This phenomenon has also been confirmed by studies on Filipino students (Bernardo, 2002; Bautista et al., 2009). Similarly, Dickson, Brown and Gibson (1984) argue that "a major source of difficulty experienced by children in the problem solving process is transforming the written word into mathematical operations and symbolization of these" (p.358). Therefore, Laborde (1990) argues that "understanding what is to be solved requires understanding the problem statement given in an oral or written form" (p.62). In this regard, a plethora of research has documented the difficulties that students encounter when solving WPs (e.g. Adetula, 1990; Badia & Armengol, 1998; Boggs, 2005; Jiang & Chua, 2010; Reed, 2001; Zhu, 2003).

WPs are mathematical problems with words. However, for a student who is learning a second (or third) language, words in that new language can create a barrier to understanding (Bresser, Melanese & Sphar, 2008). Some researchers have proposed that a major component of problem solving is the acquisition of information concerning the interpretation and use of language in WPs. Moreover, interpretations occur at two levels while understanding the problem statement. First, making sense of the language (grammar and usage of words) in which the mathematics problem was coded and next making sense of the Mathematics involved (Halai, 2001). Personal experience indicates that while working on a WP, students mainly engage in calculations regardless of understanding the problem statement.

Likewise, Tuohimaa, Aunola and Nurmi (2008) state that "children are usually asked to read (or listen to) the maths story or the problem presented, write down the mathematical operation necessary for completing the task, and then solve the problem and come up with an answer" (p.410). Learners are frequently limited to computation exercises and little or no time is spent in problem solving (Secada, 1991). Similar practices are carried out in the context of Pakistan: teachers write down the problem statement numerically for students and students arrive at the answers without focusing on how and in what way the problem is stated. When students are asked to transform the problem statement into a numeric form they merely rely on key words to do so. Contrary to this, several researchers including Leader and Middleton (2004), Montague and Applegate (1993), Polya (1957), Rickard (2005), and Ridlon (2004) suggest that students should be taught to read and understand the problem, come up with a plan, solve the problem, and then check their answer against the fact in the story problem. Researchers relate WPs to problem solving and application while students and teachers in general see WPs as nothing more than exercises in the four basic operations (Blum & Niss, 1991). Therefore, consistent with the findings of Rickard's study (2005) a teacher needs to understand the difference between problem solving and merely engaging students in routine exercises. Unless teachers realise the importance of problem solving together with language, they may continue to reinforce traditional practices such as writing down data on the board and solving WPs for students for the purpose of exercise completion. Similarly, students have a proclivity to be provided with numerical data or the key words of the word

problem regardless of understanding the meaning behind the problem statement. On the basis of previous research Oviedo (2005) asserts that most of the difficulty with WPs arises from a mismatch between text comprehension, situation comprehension, and problem solving procedures.

In short, teaching mathematical WPs does not only deal with symbols, numbers, procedures and rules; rather the language of the text is an important aspect as well. While emphasizing the linguistic aspect in solving a problem it has been recognised that "students have to use their linguistic abilities before conceptualizing a problem in mathematical terms, so that they can arrive at a correct numerical representation and problem solution" (Oviedo, 2005, p.268). Having said that, it needs to be recognised that the ability to solve a problem is not exclusively dependent on linguistic competence but also relies on students' understanding of concepts, procedures, prior knowledge and experiences (Khalid & Tengah, 2007).

3. METHODOLOGY

An action research approach was employed to help a Mathematics teacher and grade eight students understand the role of comprehension in solving WPs. The processes and issues involved in a teacher's pedagogical practices with regard to WPs were explored from a comprehension perspective. The action plan was to work on comprehension and interpretation of WPs.

The tools used for data collection were interviews, observations, field notes and personal reflections. Further relevant documents such as textbooks, the teacher's diary, lesson plans and samples of students' work were examined to enrich and triangulate findings.

Analyses occurred through a process of transcribing, coding, categorising themes and making meaning of the data. On the bases of the findings at the reconnaissance stage I planned for the first cycle and in a similar manner data were analysed to design the second cycle. Before drawing any conclusion, I kept going back and forth from the data to the research question in order to stay focused and work within the framework of the study.

4. FINDINGS

According to National Council of Teachers of Mathematics [NCTM, 1980] problem solving is the cornerstone of school Mathematics. Unless students comprehend the language of a problem statement well, the facts and procedures they know are of little use (Bernardo, 2002). This study shows that the participant teacher emphasised the role of language in WPs but downplayed the importance of explaining the problem statement. She stated:

> Language of question … yes, it is important because without understanding wording it is difficult to solve question even if students know the formula. I think, explanation of WPs is not important; we have to explain the steps at the end, it is important. (Interview, January 26, 2010)

This aspect was also evident in her teaching. While teaching the participant teacher did not focus on the problem statement. Instead, she read the problem superficially and moved to the solution without eliciting students' understanding of the problem statement. Furthermore, the teacher explained and demonstrated the solution in detail which revealed that she was solely geared towards the solution instead of being focused on the problem statement and students' understanding of it. This aspect was also evident in students' exercise books. Figure 1 depicts the solution to word problem number three that a student had reproduced from the blackboard. It appears that the teacher was not concerned about writing statements related to the word problem. Instead, she identified a mistake in the numerical solution and highlighted it with a question mark (figure 1).

On the other hand, figure 2 illustrates the solution of a word problem (the student had copied the solution from the blackboard) that did not correspond with its question. Despite this blatant mismatch between a word problem and its solution the teacher's assessment of a student's work depicted a 'tick' mark that signified that the solution was the correct one. This revealed the teacher's lack of emphasis on the problem statement; she seemed more focused on the solution instead.

Moreover, while indicating her teaching style the participant teacher elaborated:

> First of all I read the question, besides reading what I will do [sic], I will [sic] write data on the board, what is given in the question? At the end what they [students] are asking. So would come under the heading solution. (Interview, January 26, 2010)

This pedagogical practice confirmed that the teacher translated text from statements to symbols despite 'providing opportunities' to students to understand and comprehend the problem statement themselves. She was observed to transform WPs by herself without involving students in reading or understanding the text of the problem.

Instead the teacher would teach students how to interpret vocabulary and comprehend the mathematical language. Similarly, while teaching during the reconnaissance stage, I noticed that students were trying to create a numeric form of the problem by relying on key words. Thus, the inability to understand the problem statement caused errors in solving the problem. That is to say students, at times, rushed through the word problem without giving any problem solving thought to what the problem was asking (Roti, Trahey & Zerafa, 2000).

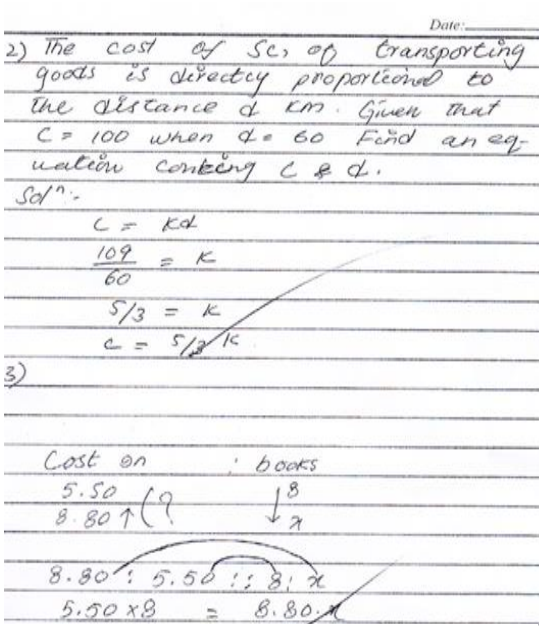

Fig.1. *Solution to word problem no. 3*

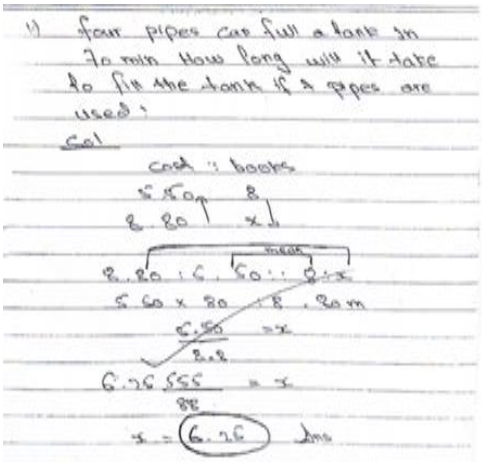

Fig.2. *Mismatch between a WP and its solution*

During the intervention stage when I asked students to try to make sense of the problem and share their understanding with each other and the class they merely stared at me. Through my observations and teaching I realised that the pupils expected me to comprehend the problem statement for them. A related vignette follows:

St.1: Ok miss, you write the data for us …
St.2: Miss, can I tell you the data?
St.3: No (angrily). *Miss likhy ge* (teacher will write it for us).
St.1: *Hann. Miss, aap data likhein* (yes miss, you write the data).
(Students 1, 2 and 3, Field notes, February, 08, 2010)

Even after the class, the teacher said:

I think it would be good and time saving if you explain on the board … what is wrong in providing data? (Discussion, February 12, 2010).

Based on classroom observations and interview, I was aware that the teacher transformed WPs into numeric form – which was referred to as 'data' – and demonstrated the solution to students who merely copied the solution from the board. Consequently, students were expecting the same from me. However, the aim was to enable the teacher to understand how to involve students in solving problems independently according to the learners' own frame of reference. Therefore, I encouraged students to try to independently comprehend WPs. It was evident that learners were not used to transforming WPs into numeric form by themselves. Therefore, the students seemed to be more focused on the key words and the direct translation of the word problem. The following excerpt reflects students' understanding of the words 'separate' and 'exceed' in a word problem (appendix 'A'):

St.1: Separate means *alag krna* (to split) and exceed means *ziyada* (more).
St.2: *Hann, separate matlab alag alag krna* (break up into parts).
St.3: *Juda krna* (to detach) and *ziyada* (more).
St.4: *Separate matlab dur krna* (separate means take apart).
St.1: *Ji nahi, dur krna kaise ho giya? Separate matlab alag krna hi hota hy bas* (separate means to split).
St.5: Exceed means more, it means one part is 7 more.
(Discussion, February 19, 2010)

The preceding excerpt revealed that students focused on the translation of individual words with an over-emphasis on certain words. Hence, they were continuously referring to the words 'separate' and 'exceed' which I think was more likely to confuse learners. Likewise, the word 'more' does not always suggest 'addition', and the word 'of' does not always indicate 'multiplication'. Further, it was also possible that a misconception of the words 'separate' and 'exceed' led students to misinterpret the text. As a result of incorrect text comprehension students were more likely to come up with faulty solutions to WPs (Nathan, Kintsch & Young, 1992). Therefore, it is suggested to keep the mathematical meaning intact while translating key words and phrases (Halai, 2004). "Problem interpretation involves translating a problem from words into a meaningful representation" (Jitendra & DiPipi, 2002, p.24). Hence, the teacher was requested to follow the students' discussion and ask what they meant. She was shown how to encourage students to draw or model the problem if learners experienced any difficulty in transforming the data to numeric form. Consequently, instead of a teacher-enforced interpretation the students were encouraged to articulate their own understanding and interpretation of WPs. Nevertheless, students were initially less inclined to accept their own abilities of interpretation and meaningful representations of WPs. This was because they were accustomed to a teacher-centred classroom where the teacher transformed and solved the problem and the learners' task was to simply copy the solutions from the board. The teacher confirmed this:

> I write data on board and then do the solution on the board but it is not always necessary [sic]; sometimes I take out the data and they themselves find solution. (Interview, January 26, 2010)

An implication is that teachers attach more importance to the product than the process – therefore, teachers and students fail to see the need for developing problem solving skills for real life scenarios. My intervention helped a teacher realise that solving WPs involved more than basic computational skills. That said, students also acquired necessary practice and developed their own abilities in comprehending WPs. They understood how the language and sentence structure contributed to their understanding of the problem statement.

5. DISCUSSION

The general findings drawn from the whole process of the reconnaissance stage and action research cycles indicate that students were not accustomed to reading and understanding WPs on their own; instead learners were dependent on the teacher to read and solve the problem for them. Contrary to this it is suggested that students should read and re-read the problem to ascertain that they have responded correctly to the question (Boggs, 2005). Along these lines, Koedinger (1991) has identified two phases that students go through when solving a story problem; the comprehension phase and the solution phase. The comprehension phase requires reading the problem logically to extract the data in order to reach the solution phase. Besides, students need to recognise which previously learned concepts, procedures, and applications must be applied to the text that they are reading. According to Ara (2007) "lack of reading skills and adequate vocabulary was a barrier for children to attempt mathematical tasks, especially word problems" (p.44). Similar findings resonated with this study – students did not consider reading the problem; rather they were more concerned with copying the solution from the blackboard. These findings suggest that providing students opportunities to read, comprehend, share and challenge each other's ideas, give reasons, argue and approach the problem from a variety of angles may result in a deeper understanding of the problem statement. The findings of the study revealed that students had not been given an opportunity to interpret WPs on their own and consequently believed that WPs could only be interpreted by the teacher. During this study, the major change observed by the end of the intervention was that students were less likely to rely on a teacher's explanation; instead, they attempted to examine and understand the problem on their own or by discussing it with peers. Besides, it was observed that such discussions provided students opportunities to clarify the nature of a problem in order to understand what was being given and what was being asked. Moreover, the findings indicate a need to challenge students' ideas and develop their skills of reasoning and convincing to solve WPs in different ways. As a result, learners would provide insights into their processes of thinking and understanding. Additionally, students develop problem posing skills which are a key indicator of deep understanding (Rizvi, 2004).

Furthermore**,** the findings of this research reveal that students seemed to have difficulty in focusing on the relevant information provided in WPs. For example, learners sometimes focused only on the numbers while at other times on the words. Therefore, it is essential to facilitate students in identifying the information from the WPs so that their metacognitive knowledge develops. Overemphasis on any one aspect is more likely to confuse students. A similar scenario was experienced during this research study where students were confused due to overemphasis on certain words such as 'separate' and 'more'. Moreover, focusing on key words "subverts mathematical understanding" and often leads to wrong answers (Clement & Bernhard, 2005, p. 364). For example, the word 'more' does not always imply 'addition'; its meaning depends on the way it is used. Hence, giving more weightage to key words leads to partial success as it falls short of solving more complex problems where the meanings of words differ. Consequently, the findings of this research reveal that the emphasis on key words to arrive at the correct representation and solution of problems can be counterproductive and may reside with the students as an incorrect

strategy that may backfire in higher grades (Marshall, 1995). Besides, certain words and phrases might be ambiguous to students in an ELL context and the use of such terms in WPs leads to incoherent representation. Therefore, it is crucial for teachers to assist students in understanding the utility of key words and their meanings in context. Moreover, teachers need to elicit from students the latter's understanding of words and phrases. These issues can be overcome if students are offered alternative strategies for tackling mathematical word problems.

6. RECOMMENDATIONS

Recommendations offered are the result of reflection on the findings and analyses.

- Opportunities need to be provided by teachers for students to demonstrate their mathematical understanding in various ways such as illustrations, discussion, arguing and reasoning.
- Efforts need to be made to help teachers realise the importance of comprehending word problems.
- Further research is needed to determine, more clearly, which factors influence students' ability to comprehend WPs and identify the best practices for teaching ELLs to solve WPs meaningfully.
- Teacher education programmes should examine the efforts of other countries (e.g.Singapore, Hong Kong, England and The Netherlands) regarding the best problem solving strategies for development and implementation of a successful Mathematics curriculum in Pakistan.
- Teachers need to provide opportunities for students to personalise WPs by using content and language that is familiar to the learners.
- Teachers and teacher educators need to have a greater awareness of language issues in the teaching and learning of Mathematics.
- The importance of cognitive, affective, metacognitive and social strategies need to be understood to facilitate learning by making students aware of their own mental processes and by providing direct instruction in thinking and learning strategies (Jones & Idol, 1990).
- Teachers and teacher educators need to initiate small changes at the classroom level that would lead to positive differences in students' lives. For instance, by helping students to develop conceptual understanding of concepts rather than employing rote memorisation strategies.

7. CONCLUSION

This study discussed the issues related to teaching WPs in an ELL context which is supported by literature and evidence. The study aimed to develop a teacher's understanding regarding the role of comprehension in solving mathematical WPs. It was found that a transformational approach is less effective as it leads students to become more product-oriented. Therefore, student-oriented teaching is better suited for supporting ELL students to understand WPs. Nevertheless, teachers and students need to experience less linguistic complexity to comprehend the problem situation and describe solution strategies. Thus, WPs could be taught meaningfully and systematically by ensuring that different aspects such as language, concepts and content are given due consideration during teaching and learning.

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APPENDIX 'A'

ACTIVITY SHEET

Name_____ Date: February 19, 2010

Separate 71 into two equal parts such that one part exceeds the other by 7.

UNDERSTAND THE PROBLEM
Read the problem and write down the information.

Words:
_____
_____

Numbers:
_____
_____

What is known?
_____
_____

What are we looking for?
_____
_____

Restate the problem in your own words?
_____
_____

Would a diagram or picture help?
_____
_____

DEVISE A PLAN
What relationships exist between the known and the unknown?
_____
_____
Write an equation/expression/formula.
_____
_____

CARRY OUT THE PLAN
Solve the equation.
_____
_____
Check each step. Do you need to re-evaluate your plan?
_____
_____

EXAMINE THE SOLUTION
Read the problem again to see how the solution of the equation relates to the question.
_____
_____

Verify the solution with the words of the problem, not your equation.
_____
_____
_____